STATIONARY DISPLACEMENT OF A BODY BY A SHOCK WAVE

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There is examined the possibility of stationary motion of a quite streamlined body together with a shock front in a gas in which the body is contained as a float in water (which is substantially different from the thrusting action of a wave on the obstacle around which it flows).

The emergence into this mode can be distinct and it has no general description (the secondary jolt on the body before the arrival of the wave, the pulse from the wave itself, etc.).

The body motion relative to the initial gas will be supersonic, and oblique compression and rarefaction waves start from it. Their role can be taken into account for the case of a float shaped like a wing with a narrow rhombus profile. The motion diagram is shown in Fig. 1, where the solid line denotes the main shock front, and the dashes are the weak compression and rarefaction waves (characteristics) from the profile.

The motion is stationary in a coordinate system coupled to the float, and the gas flows to the left at a supersonic speed $D > c_0$.

A shock emerges from the wedge vertex at the speed of sound c_0 relative to the gas in front. The tangential velocity is conserved on the wave, i.e.,

$$D \cos \beta = u \cos (\beta - \alpha).$$

The normal velocity changes by the quantity

$$v_n = D \sin \beta - u \sin (\beta - \alpha) = D \sin \beta [1 - \mathrm{tg}(\beta - \alpha)/\mathrm{tg}\alpha].$$

The pressure increases by

$$p_1 - p_0 = \rho_0 D \sin \beta \cdot v_n = \rho_0 D^2 \sin^2\beta [1 - \mathrm{tg}(\beta - \alpha)/\mathrm{tg}\alpha].$$

Substituting $D = Mc_0$ (M is the Mach number for the fundamental wave)

$$\rho_0 c_0^2 = \gamma p_0, \quad \sin \beta = 1/M,$$

we obtain for small α after manipulation

$$p_1 = p_0 \left(1 + \frac{\gamma M^2}{\sqrt{M^2 - 1}} \alpha\right).$$

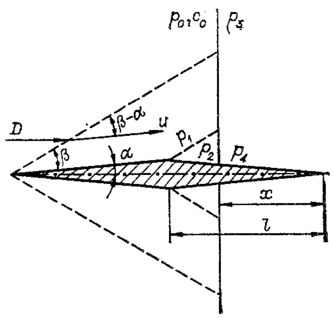

Fig. 1

Chelyabinsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 135-137, March-April, 1980.

Let us write the pressure in other domains. We obtain

$$p_2 = p_0\left(1 - \frac{\gamma M^2}{\sqrt{M^2-1}}\alpha\right)$$

analogously to the method used to obtain p_1. The pressure behind the main wave, expressed in terms of its velocity (or the Mach number) has the form $p_3 = p_0\,(M^2\,(h+1) - 1)/h$, where $h = (\gamma + 1)/(\gamma - 1)$.

A pressure p_4, different from p_3 by a quantity on the order of $p_2 - p_0 = -p_0\gamma M^2\alpha/\sqrt{M^2-1}$, because of perturbation by weak oblique waves, is given by

$$p_4 = p_0\frac{M^2(h+1)-1}{h} - kp_0\frac{\gamma M^2}{\sqrt{M^2-1}}\alpha,$$

where k is a number of the order of one (for weak waves $k=1$). The equilibrium condition for the rhombus in the motion direction is $p_1 l\alpha - p_2(l - x)\alpha - p_4 x\alpha = 0$, from which $x/l = (p_1 - p_2)/(p_4 - p_2)$ or

$$\frac{x}{l} = \frac{\dfrac{2\gamma M^2}{\sqrt{M^2-1}}\alpha}{\dfrac{M^2(h+1)-1}{h} - 1 - (1-k)\dfrac{\gamma M^2}{\sqrt{M^2-1}}\alpha}.$$

Since $\alpha \ll 1$, then the terms with αB in the denominator can be discarded, after which we obtain as a result of simplification

$$\frac{x}{l} = \frac{(\gamma+1)M^2}{(M^2-1)^{3/2}}\alpha.$$

By diminishing α it is always possible to make $x/l < 1$, which will indeed denote the possibility of stationary motion. Upon random loading of the rhombus in a wave greater than at x (including even a somewhat deeper middle) it will push out backward, while under random floating it will be submerged downward.

The wing can be released from side loading by fitting it into a ring around the horizontal axis. Let us note that the question of the build-up of random vibrations, i.e., vibrational instability, still remains unexplained.

The stationary motion conditions are satisfied comparatively easily; for instance, for waves in air with an excess pressure of 1 atm ($p_3 = 2$ atm, $M^2 = 13/7$) and $x/l = 5.6\,\alpha$ is obtained, i.e., a rhombus with $\alpha < 1/5.6 \approx 10°$ is suitable.

Let us note that the start of the ring will be easier, the smaller its density, and its weight is not essential for stationary motion.